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CHANCE CONSTRAINED PROGRAMMING METHODS IN PROBABILISTIC PROGRAM--ETC(U)
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The University of Texas
Austin, Texas 78712

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Research Report CCS 427

CHANCE CONSTRAINED PROGRAMMING METHODS IN PROBABILISTIC PROGRAMMING--A RESPONSE TO A.J. HOGAN, J.G. MORRIS & H.E. THOMPSON, "DECISION PROBLEMS UNDER RISK AND CHANCE CONSTRAINED PROGRAMMING: DILEMMAS IN THE TRANSITION," MANAGEMENT SCIENCE 27, NO. 6, JUNE 1981.

by

A. Charnes
W.W. Cooper

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CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Business-Economics Building, 203E
The University of Texas at Austin
Austin, Texas 78712
(512) 471-1821

1. INTRODUCTION

This is a response to the article "Decision Problems Under Risk and Chance Constrained Programming: Dilemmas in the Transition" [25] in which Professors Hogan, Morris and Thompson (HMT hereafter) recommend abandonment[1/] of Chance Constrained Programming (=CCP) in favor of Stochastic Programming with Recourse (=SPR)--which we shall also refer to as 2-stage Linear Programming Under Uncertainty (=LPUU) since this is the main variant of SPR which is relied upon for these conclusions in [25]. In the interest of clarity and brevity, we do not pursue all of the topics covered in [25] since, as will become evident, a rather lengthy response is required to chase down even major issues. We also believe that [25] is directed to conceptual rather than practical issues of application and so, also for brevity, we brush aside qualifiers that appear in statements like the following: "We wish to emphasize that recourse problems characterize almost all [sic] real decision problems involving risk." Except for possibly affording some degree of protection to HMT, we do not see that such qualifiers serve any useful purpose. In a similar vein we ignore other literary artifacts and devices such as the use of quotation marks which supposedly indicate a special meaning for commonly used terms, but which are employed repeatedly without explicitly indicating what special meaning is intended.[2/]

Proceeding in this manner we now set forth the following criticisms of the HMT article [25].

1. It contains erroneous claims of conceptual and/or mathematical equivalences between different types of models based on the supposition that such equivalence is established between two models when optimal solutions of one can be used to obtain optimal solutions to the other.

2. Focusing on the class of zero order decision rules, there is a failure to recognize that CCP has many forms, e.g., with individual, joint, total or conditional chance constraints, as well as many classes of statistical decision rules and many possible objective functions. In fact, the 2-stage LPUU model, in favor of which HMT recommend abandonment of CCP, is itself a special case of one class of CCP models.

3. There is a failure to recognize that the value of information is a function of the model employed (including its decision rules) and one cannot validly compare values of information between two different models (which embody different concepts and information structures) much less use this concept as the basis for choice between models without also considering what information is available and how it is to be used.

1/ Or, at best, allowing it some degree of tolerance until certain (promised) computer codes become generally available.

2/ See, for instance, the repeated use of "equivalence" in [25], leaving the reader to infer what is intended--perhaps from references to a variety of other articles--and which we accord the usual mathematical meaning of "isomorphism."

4. Consideration of CCP as a practically rooted method for planning in situations where information is not available for all possible responses is pushed aside on the supposition that, at worst, such complete information may only be "costly" to obtain for the uses prescribed in [25] while, at the same time, the work of Prékopa and his associates[1] is cited as advancing computational alternatives to CCP even though *the modeling is via CCP with* joint chance constraints.

5. Citations are offered *from* papers by others while ignoring materials *in* these same papers which contradict or qualify the interpretations that are effected in [25]. Also ignored are large parts of the theoretical literature which are directly pertinent to the issues in [25].

2. MODEL EQUIVALENCE

To highlight what we are saying about model equivalence we might begin with the dual problems of linear programming (L.P.). Such problems generally have optimal values and these are equal to each other. Also, as is well known, one of them may be used to solve the other--e.g., via the simplex method.[2] Neither mathematical nor conceptual equivalence is thereby established between the constructs of the dual problems.

For illustration we turn to the L.P. duals used for plastic limit analysis modeling, as discussed in [8],[3] since this is one of the few cases in which the dual variables have been interpreted over their full range, and not merely at an optimum. In this class of cases the variables in one problem represent internal stresses in the members of a structure (e.g., a truss) while the variables in the other problem represent virtual displacement velocities at the joints. However, stresses and velocities are different physical concepts and so, although the dual problems are related to each other they are neither mathematical nor conceptual equivalents even though one problem may be used to provide optimum values for the other.

In [23], Garstka defines[4] "equivalence" explicitly so that its usage is restricted to situations in which a CCP model may be formed to obtain solutions to a corresponding SPR model and vice versa. Ostensibly relying on Garstka's analysis, HMT in [25] proceed, nevertheless, to use "equivalence" in a great variety of ways, up to (and including) full-scale mathematical equivalence. This is done, we may add, despite the fact that Garstka concludes in favor of conceptual non-equivalence and in his paper with Wets [24] he shows SPR to be a special case of CCP!

1/ The confusion in [25]--in a reference specifically cited by HMT--is surprising in that at the outset of [33] the authors specifically note that the model name, "STABIL," is not an abbreviation but arises from the fact that "the model contains a probabilistic constraint which prescribes a high probability level [chosen near unity in practice] under which the system must operate." We shall have more to say, below, about the seemingly extensive but casual review of the literature on applications of CCP that is to be found in [25].

2/ Such choices of solution methods have been called "algorithmic completion of a model" in the treatment of modelling strategies covered in [7].

3/ See pp. 646-650.

4/ Consideration is also confined to 2-stage LPUU and CCP with zero order decision rules. See pp. 84-85 in [23].

Although citing our exchange with Blau[1/] as providing the major motivation for their discussion, HMT fail to mention that we make specific note of the unsuitability of the exclusive use of zero order decision rules for the uses they wish to make of them and the non-equivalence of the information structures that Blau provides for his CCP and SPR decisions.[2/]

Conceptually, CCP provides for wide ranges of model constructions from different classes of decision rules and chance constraints. It has not always been computationally practical, however, to employ intuitively desirable classes of nonlinear decision rules or joint chance constraints because of the mathematical problems involved. In probabilistic PERT cases, for example, the probability algebra results in unmanageable representations of the distributions of random variables. Even from the outset, individual rather than joint chance constraints were employed from manageability considerations with recognition (as also with the use of simpler classes of decision rules) that this could lead to overly conservative actions.

Such uses can, and should, be checked in a variety of ways. For instance, in the heating oil problem for which CCP was originally developed, a check of the company's scheduling practices showed that linear decision rules were practicable as attested to by their use in company practice. Furthermore, the CCP model using this class of decision rules was tested against the schedules that would have resulted from company practices. These tests even extended to situations in which the company scheduler was given "perfect information" in the form of the actual demands that would materialize on each day of the heating oil season but access to this information did not succeed in improving cost performance compared to uses of the model wherein these same data were presented at random.[3/]

Of course progress has been made (and continues to be made) so that new alternatives are now open and presumably still others will continue to open in the future. The pressure of problems as well as the alternatives available for addressing them has generally played an important role in these developments. For instance, in a case involving problems of differing sizes of dams in the design of a system of reservoirs, Prékopa and Szantai

1/ See [1] and [4].

2/ We are referring to the kinds of models and probability distributions used by Blau and by HMT. Conditional chance constraints may also be used in which the risk conditions are formulated in terms of distributions conditioned on sample information. See, e.g., [15] and [31]. Zero order rules in which the constraints are conditional on sample information may also be employed. See [15], [28] and [30].

3/ This should make clear that information value (including perfect information) cannot be judged separately from the models (and decision rules) used and, indeed, it is possible that information improvement may worsen performance when utilized in some models (e.g., ones using inadequate or erroneous decision rules).

in [35] were led to tackle the difficult joint chance constraint analyses and computation problems because the use of individual chance constraints were judged to yield results that were too conservative.1/

3. RISK EVALUATION AND INFORMATION AVAILABILITY

Another major criticism of CCP offered by HMT is that the usual models do not seem to evaluate explicitly--i.e., choose in a "rational manner"--what HMT might refer to as optimal values for the probabilities of satisfying the chance constraints. Concomitantly, HMT appear to assume that no such representations are even conceptually possible. This assumption is false, however, as witness the following example which illustrates one way2/ in which such a determination could be modeled:

$$\min E(c_1 x_1 + c_2 x_2(b)) - w_1\alpha_1 - w_2\alpha_2$$

subject to

$$(1) \qquad P(A_{11}x_1(b) \geq b_1) - \alpha_1 \geq 0$$

$$P(A_{21}x_1(b) + A_{22}x_2(b) \geq b_2) - \alpha_2 \geq 0$$

$$D_1\alpha_1 + D_2\alpha_2 \geq \gamma$$

$$x_1(b),\ x_2(b_1) \geq 0,$$

where $P(\ldots)$ refers to the probability of occurrence of the parenthesized expression and E means "expected value." For our needs here we shall employ individual rather than joint chance constraints and we shall also take the probabilities to be total rather than conditional. These probabilities are then defined relative to the random vectors c_1, c_2 and b_1 and b_2 and the vectors α_1 and α_2 which have probability measures as their components. The α_1 and α_2 choices (which are now variable), are to be effected by reference to the weighting vectors w_1 and w_2 in the functional, subject to conditions imposed on these choices by the relations associated with the matrices D_1 and D_2 and the vector γ, all with known constants for their elements. Finally, the A_{ij} are also matrices of constants and the chosen $x_1(b)$ and $x_2(b)$ designate stochastic decision rules which involve the random

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1/ See also [34].

2/ Other ways of evaluating (and choosing) the risks to be entertained in a CCP context may be found in [35] and [34]. See also [32].

vector pair $b = (b_1, b_2)$ usually according to "informational feasibility"1/ (i.e., according to knowledge of sample values at the time of decisions $x_1(b)$ and $x_2(b)$).2/

We note parenthetically that the system (1) also exhibits, incidentally, a 2-stage CCP problem for which the 2-stage LPUU form is the special instance in which $x_1(b) = x_1(b_1)$ where b_1 is a constant vector, α_1, α_2 have all components set at unity with the D_1 and D_2 matrices and vector γ set at zero and $x_2(b) = x_2(b_2)$.

There are mathematical and computational3/ problems to be addressed, to be sure, but these are less important than the information difficulties which would be confronted in implementing such a model in anything like the manner suggested by HMT. As a case in point we might mention that a multi-stage variant of the above model was developed as part of the work we undertook with D. B. Learner (and others) at BBDO, Inc., on the DEMON and NEWS series of models which were to be used in aiding that firm's clients in developing new product marketing strategies.4/ This part of the work was abandoned, however, when potential clients displayed no enthusiasm for meeting the kinds of information requirements involved.

Returning to computational problems arising from explicit incorporation of α variables as in (1), it is much simpler, easier and direct to posit α values and perform sensitivity analyses. The extensive experience of many persons in the use of dual evaluators in ordinary linear programming makes it hard to understand why HMT think so little of efforts pointed in these directions.5/ Another purpose of such developments is to take advantage of CCP formulations in which information requirements are restricted only to small (tail) portions of the distributions. A step-by-step evaluation procedure might then be used to ascertain whether additional information collection might be justified. In fact the dual problems--zero order variety--presented on p. 25 in [6] were developed to provide simultaneous access to both c and b vectors when conducting such analyses.

We don't mean to imply that such approaches will always work, even when the use of zero order rules is applicable. As a case in point, we might mention the study we undertook for the U.S. Coast Guard with J. Harrald, K. Karwan and W. A. Wallace which, as reported in [14], was directed to developing models for use in budgeting and positioning equipment to deal with large (tanker) spills in U. S. coastal waters.6/ Because these decisions had to be made in advance of such events, a use of zero order

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1/ See [15], [19] and [30] for further discussion.

2/ We may also allow for some of these to be forecast values as in e.g., the work being done by Hsu [26] in applying CCP to time series analyses under chance constraints to ensure compliance with various policies.

3/ One part of an approach that might be used is set forth in the article [3] which we wrote with R. F. Byrne and K. Kortanek.

4/ The idea was to utilize lost-profit-opportunity weights in the functional as an alternative to the ubiquitous use of payback period constraints to guard against the risk of an even more profitable opportunity subsequently appearing while funds were tied up in the (new) product being considered for profitable introduction into the market. See [9], [10], [11] and [12].

5/ It remains to be seen whether the computer codes promised in HMT will yield similarly valuable uses for evaluations when they become available.

6/Including inland waterways where such spills might also occur.

rules was indicated. Here the information was (again) meager, especially in regard to the tails. This frustrated the kind of "tail only" considerations of CCP which had been advantageously employed in other contexts and we therefore resorted to a mixture of CCP and goal programming.1/ Even this did not succeed (for various reasons) but at least a start was made. Following the advice of HMT, as given in [25], would have required waiting until an adequate number of sinkings occurred--a situation which the Coast Guard is seeking to prevent--and we do not think that this offers a good way to advance either science or the practice of management.

4. AN EXAMPLE

As was the case in Blau [1], the center piece of the analysis in [25] involves a very simple example provided by HMT as a sort of counter example to all of the CCP that we have been describing (and more besides). To make sure there is no misunderstanding,2/ we therefore reproduce this example and its faulty analysis from [25], p. 701, as follows:

> "Even though a CCP model produces the same solution as an "equivalent" recourse complete SPR model it still may be incomplete if it miscalculates EVPI [= Expected Value of Perfect Information]. Consider an example adapted from [1] whereby a retailer must choose an inventory level for a given product for a given period. Assume the decision maker has determined that it is best to satisfy all demand. This means that the revenue from the retail operation is not controllable but the costs are. Because it is less "costly" to fill orders on recourse than to lose a sale immediate recourse action is taken to see that demands not filled out of normal inventory will be filled within a very short time--before the next period's ordering takes place. Letting ℓ be the cost of backfilling an order we may write the retailer's recourse problem as
>
> $$(2) \qquad z^R = \min \{x + \ell \int_x^\infty (r - x)dF(r) \mid x \geq 0\}$$
>
> where x is the number of units to be stocked, r is the demand during the period under consideration and F is the c.d.f. of r with $P(r = 0) = 0.9$ and $P(r = 1) = 0.1$ assumed known. The "normal" inventory stocking cost is 1 and $\ell > 1$. Setting the derivative of the convex function of x to zero gives
>
> $$(3) \qquad x = F^{-1}(1 - 1/\ell) \text{ where } F^{-1}(\alpha) = \sup_x\{x \mid F(x) < \alpha\}.$$
>
> "This implies that the demand is <u>not</u> filled from normal inventory with probability $1/\ell$. For $1 < \ell \leq 10$ the optimal solution to (1) is $x = 0$ and $z^R = .1$. For $\ell > 10$ the optimal solution is $x = 1$ and $z^R = 1$. The SPR problem can be converted to a mathematically equivalent CCP problem by letting $\alpha = 1 - 1/\ell$ and writing

1/ See [14]. We might also note that recourse to goal programming was suggested by earlier (successful) efforts in developing a goal programming model for use by the Coast Guard in dealing with its budgeting efforts for the small-spill problem. See [13].

2/ E.g., the kinds of misunderstandings and misperceptions which HMT impute to us in our discussions of Blau [1].

$$\text{(4)} \qquad z^C = \min\{x \mid P(x \geq r) \geq \alpha;\ x \geq 0\}$$

or

$$\text{(5)} \qquad z^C = \min\{x \mid x \geq F^{-1}(\alpha);\ x \geq 0\}."$$

Although the above example forms a center piece of the analysis in [25], the referees for Management Science apparently did not bother to check the development either for ambiguities or error. That this should have been done may be made clear by our explicit calculation of (2), noting that the integral is just $E_r\left(\frac{|r - x| + r - x}{2}\right)$, to obtain,

$$\text{(6)} \qquad z^R(x) = \begin{cases} x(1 - 0.1\ell) + 0.1\ell, & 0 \leq x < 1 \\ x & , \quad 1 \leq x. \end{cases}$$

For HMT's "convex function," $z^R(x)$, the derivatives are therefore

$$\text{(7)} \qquad \frac{dz^R}{dx} = \begin{cases} 1 - 0.1\ell & 1 \leq x < 1 \\ \text{does not exist for} & x = 1 \\ 1 & 1 < x \end{cases}$$

Thus, the derivative cannot be set equal to zero as HMT prescribe unless $\ell = 10$, in which case every value of x between zero and one has $\frac{dz}{dx} = 0$.

The air of déjà vu in going from Blau [1] to HMT [25] makes us wonder whether the criticisms should be levelled at Management Science for its refereeing rather than these authors. Ideally the referees should be familiar with CCP and related literatures so that they might be able to alert the authors on the need for limiting their discussion of such simple examples relative to the rich array of alternatives that are available. Failing this, they should at least be able and willing to check for elementary errors and ambiguous formulations and developments.1/

The formulation in (2) also jumbles together a variety of concepts such as objectives, policies, criterion elements and constraints2/ that had best be treated separately in the kinds of analyses and interpretations attempted by HMT. For instance, as in the heating oil problem, a policy assumes a form in which exceptions may be made provided they are not too large or too frequent. In a management context this means that executive attention is required to ensure that the admitted deviations do not attenuate the policy by virtue of their frequency or magnitude. This distinguishes a "policy" from a "rule" which holds without exception and hence can be administered at more clerical levels.3/

1/ In Blau's case the referees could have called his attention to the way he had altered his problem and thereby spared him from some of our criticisms as well as the explanations offered by HMT on p. 704 in [25]--we are not sure which Blau would prefer--which assume strange (almost bizarre) forms in which Blau is asserted to have had different problems in mind "lurking behind" those he articulated and which HMT extend even further to the invention of a mythical retailer who is said to have views of the problem which differ from those of Blau, even though the problem and its interpretations all involve only Blau in much the same manner as the problem we have just quoted (above) from HMT.

2/ See Chapter I ff. in [8].

3/ For instance, an academic committee may be required to consider exceptions to a policy that students are required to maintain at least a B average to remain in a particular school whereas clerical routines may suffice to ensure that all fees are paid before a student receives a diploma.

We illustrate within the limits of what the above very simple (indeed, overly simple) example will allow by reformulating the problem as follows:

$$\min x + \ell \underset{r}{E}\, y(r)$$

subject to

$$\begin{aligned} & P(x \geq r) \geq \alpha \\ (8.1) \qquad & x + y(r) \geq r \\ & x,\ y(r) \geq 0. \end{aligned}$$

where y(r) is to be chosen after r is observed whereas x is to be chosen, in accordance with the first constraint, before the value of r is known. Because of the minimization we can then take y(r) as

$$(8.2) \qquad y(r) = \max\ \{r - x,\ 0\} = \frac{|r - x| + (r - x)}{2}.$$

Evidently the last constraint in (8.1)--which is a "probability 1" constraint-- involves choices of y that are to hold in this case without reference to the specific criterion values in the objective. The first constraint is the one where judgment is to be exercised relative to the degree of flexibility admitted by the α choice, which here reflects a policy (for whatever reason it was adopted)[1] of undertaking anticipatory buying which will suffice to meet subsequently realized demand at least α proportion of the time.

Ideally a policy of the latter kind should be varied and iterated by introducing additional constraints to provide guidance on the frequency with which different deviational magnitudes should be tolerated. We shall not pursue that topic here,[2] however, in order to maintain contact with the development in [25]. Using (8.2) we replace (8) with our usual form, as in [18],

$$(9) \qquad \begin{aligned} & \min x + \ell \underset{r}{E} \left(\frac{|r - x| + (r - x)}{2} \right) \\ & \text{subject to} \\ & \quad x \geq F^{-1}(\alpha) \\ & \quad x \geq 0, \end{aligned}$$

1/ HMT seem to think that an objective should always take precedence over the policy constraints. It is for this reason that we have inserted the parenthesized expression. A fuller discussion may be found, in a goal programming context, on pp. 45 ff. in [27].

2/ See [6] for further discussion.

where, as in [25], we are using the zero order rule and assuming that F is the c.d.f. with $P(r = 0) = 0.9$ and $P(r = 1) = 0.1$.

With these assumptions the model (9) becomes

$$\min x + 0.1\ell\left(\frac{|1 - x| + (1 - x)}{2}\right)$$

subject to

$$x \geq F^{-1}(\alpha) \tag{10}$$

$$x \geq 0,$$

or

$$\min z(x)$$

subject to

$$x \geq F^{-1}(\alpha) \tag{11}$$

$$x \geq 0$$

where $z(x)$ is the objective function in (10).

We now observe that for $0 < \alpha < 0.9$ the first constraint in (11) requires merely $x \geq 0$ and coincides with the second one. For $0.9 \leq \alpha \leq 1$ the first constraint requires $x \geq 1$ and makes the second constraint redundant. The optimal values $z(x^*)$ may therefore be recorded as in the following tabulation:

(12)

	$z(x^*)$ $0 \leq \alpha < 0.9$	$0.9 \leq \alpha \leq 1$
$1 < \ell \leq 10$	0.1ℓ	1
$10 < \ell$	1	1

Thus we record 4 possibilities for optimal solutions rather than only the two recorded by HMT. The loss of solution possibilities may be an important consideration, e.g., for the kinds of explorations and evaluations considered by HMT, and hence should not be overlooked when following a model replacement strategy en route to a solution. See, e.g., pp. 197-198 in [8].

Next we observe from the first row of (12) that there is an expected incremental cost of $1 - 0.1\ell$ in going from $0 \leq \alpha < 0.9$ to $0.9 \leq \alpha \leq 1$. This, however, is not an expected cost of information since the information structure is the same in both cases. It is rather the expected cost of altering the α policy from one level to the other.

If perfect information means that one knows r when ordering x then $x^* = r$ and the minimum cost is

$$(13) \qquad Ex^* = Er = 0.1.$$

With this "perfect information" in hand the α policy becomes superfluous with an incremental benefit of $0.1(\ell - 1)$ in the first cell and $1 - 0.1 = 0.9$ in the other 3 cases over the α policy.

The superfluity referred to in the above example arises because there is no ready meaning that can be assigned to a policy of "anticipatory buying" when perfect information is available. We should add that it is not unusual for this assumed perfect information state to change the character of the problem. For instance, in the Coast Guard's large spill problem, which we referenced in the last section, the problem would change from containment of the damage from any large spill that might occur to prevention of the spill before it occurred.1/ In the case of the DEMON models, for marketing new products, the availability of perfect information would change the problem from one of selecting the studies to be conducted to a problem of deciding (at once) on what products to market and how they should be marketed. In each of these cases (and others, too) the problems to be addressed arose precisely because of lack of information so that an assumed availability of perfect information amounts to assuming the problem away. Thus, in contrast to HMT who assign EVPI (expected value of perfect information) a role of "eminent importance in applications" we prefer to assign this role to information that is feasible to obtain which, in the terminology of CCP, is called "operationally feasible information."2/

Following the route which HMT prescribe also runs the risk of overlooking matters of more importance than anything that their example admits. For example, there is the need for providing for and evaluating conditions beyond the planning horizon incorporated in a model. The kind of problem considered by HMT, as above, assumes a repetitive operation of the same kind carried into an indefinite future. This does not lend itself to illustrations of the kind we are considering and so we shall have to content ourselves with only a general verbal description.

1/ The latter being the more important problem was not tackled at the time only because of lack of information sufficient to estimate even the probability of major spill incidents at each possible point of occurrence. Indeed, it was the absence of information on these spill probabilities that dictated the choice of the containment problem as the focus of this work.
2/ See [15] and [19].

Information on beyond-the-horizon events and possibilities is generally much harder to come by with any degree of reliability than is the case for within-the-horizon information. With this in mind we have elsewhere developed and described an approach via the use of "horizon-posture constraints" so that boundary conditions on beyond-the-horizon possibilities can be evaluated for their consequences on present and projected (within horizon) operations and plans.1/ Indeed, without such constraints, even in non-stochastic problems, supposedly optimal within-horizon plans may have absurd posture consequences (e.g. zero inventory positions) because the model is not told that the world and/or the business does not end at the stipulated horizon date.

5. USES AND APPLICATIONS

There is a refrain that runs through [25] which is perhaps best summarized in the following quotation2/: "It is apparent...that CCP users, including its originators, are using CCP not to simplify the stochastic recourse problem, but..., etc." Apparently HMT believe that CCP was developed only to relieve part of the burden of SPR users.

In the mid and early 1950's, when we were developing Chance Constrained Programming with G. H. Symonds (and others) in the context of scheduling heating oil at Esso, we were not aware of other alternatives (or at most we were only dimly aware of them) such as those being developed by G. B. Dantzig[20] and G. Tintner [36]. We were more immediately concerned with development of the kinds of constraints and modeling possibilities that could handle the multiple inequalities needed to deal with policies and operating conditions <u>as they were to be found in the company's scheduling practices.</u>3/ Indeed, the name Chance Constrained Programming was coined only subsequently and used in the title of [5] to distinguish this approach from others such as are to be found in the work of G. B. Dantzig [20] and G. Tintner [36]4/ To comprehend all of these developments while retaining their separate identities we suggested (in 1965) the term "probabilistic programming"--as in the title of this paper.5/ We saw no reason then, and we see no reason now, why any part of these separate efforts or their various possible combinations and extensions should be abandoned--as is advocated in HMT. Such an abandonment, we might add, overlooks the use that has been made of these models in bringing in a rich array of actual problems as a source of management science research (and teaching and text material) as well as their use in providing a variety of alternatives for addressing such problems--or addressing different aspects of the same problem in different ways--in management science practice.

1/ See [2] and [16].

2/ From [25] p. 713.

3/ I.e., as distinct from formally expressed <u>resolutions</u> at, e.g., the board of directors or other top-management levels of the company.

4/ Somewhat curiously, neither Dantzig nor Tintner are referenced in [25], which oversight may possibly be due to the great amount of naming and renaming that has gone on in this literature since their initiating work was published.

5/ This suggested name was offered in the paper we presented at the first world Econometric Society Meetings in Rome in 1965. See also [15].

HMT announce near the outset of [25][1/] that "There is no intention to depreciate the quality of the research within the received tradition of chance constrained programming by the authors whose works are mentioned.... Rather, we [HMT] wish to cause a rethinking of the received tradition." We have just addressed some comments to the second sentence and we now close with some summary comments on the first.

We are not familiar with all of the applications cited in [25], but their comments on papers with which we are familiar make it appear dubious that HMT have considered them in any depth. For instance, on p. 712 in [25], HMT fault Kirby [30] for saying that some of the chance constraints employed in the heating oil model at Esso allowed storage capacity "to be exceeded a very small proportion of the time." This is not the absurdity that HMT indicate it to be. As might be expected in a study guided by G. H. Symonds--then the Chairman of Esso's Manufacturing Technical Committee--careful attention was paid to company policy and practice which (at the time) was simply to run such excess out into fields available for use (on such rare occasions) and then to burn it as not being suitable for further use as heating oil. The resulting lack of availability of the heating oil consumed in this manner could also have been handled by adding additional model details but the probability of occurrence of this event was so small as to make this not worth doing.

Other comments in HMT's discussions of work by others are misleading as well as inadequate. As a case in point we might refer to statements on p. 712 of [25] in which HMT assert that we treat liquidity requirements by SPR in our article with Byrne and Kortanek [2] but fail to note that we also treat other aspects of the problem of liquidity maintenance by CCP. By this omission HMT conceal from readers the important possibilities offered by CCP-LPUU combinations for treating different aspects of risk associated with different aspects of liquidity in the same problem.[2/]

We do not propose to chase down further examples of this kind. Instead, we summarize by saying that the view of "applications" and "decision makers", etc., in [25] is a curious one--and a far cry from usages that would pass muster for Gene Woolsey's Interfaces.

1/ Footnote 1 p. 698.

2/ Including the possibilities for dual evaluators that allow negative adjustments to the discount rates that would otherwise be used, e.g., to allow for reductions in portfolio risks which may concurrently occur with the addition of risky items to the portfolio type payback period constraints used in [2]. For another CCP-LPUU usage combination see [16]. See also [32].

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